Final Report for Period: 07/2007 - 06/2008
Submitted on: 11/14/2008
Principal Investigator: Thomas, Robin .
Award ID: 0354742
Organization: GA Tech Res Corp - GIT
Submitted By:
Thomas, Robin - Principal Investigator
Title:
FRG: Collaborative Research: The Four-Color Theorem and Beyond

Project Participants

Senior Personnel

Name: Thomas, Robin
Worked for more than 160 Hours: Yes
Contribution to Project:

Post-doc

Name: Oum, Sang-il
Worked for more than 160 Hours: Yes
Contribution to Project:

Graduate Student

Name: Norine, Serguei
Worked for more than 160 Hours: Yes
Contribution to Project:
Graduate research assistant

Name: Asadi, Arash
Worked for more than 160 Hours: No
Contribution to Project:
Graduate Research Assistant

Name: Hegde, Rajneesh
Worked for more than 160 Hours: No
Contribution to Project:
Graduate Research Assistant

Name: Nejedly, Pavel
Worked for more than 160 Hours: No
Contribution to Project:
Graduate Research Assistant

Name: Postle, Luke
Worked for more than 160 Hours: No
Contribution to Project:
Graduate Research Assistant

Name: Wollan, Paul
Worked for more than 160 Hours: No
Contribution to Project:
Graduate Research Assistant

Undergraduate Student

Technician, Programmer

Other Participant

Research Experience for Undergraduates

Organizational Partners

Other Collaborators or Contacts

Daniel Kral, Fulbright Fellow

Activities and Findings

Research and Education Activities:

The main research activity is basic mathematics research. The results are disseminated through scientific publication, lecturing both at conferences and scientific meetings as well as giving public lectures and teaching minicourses.

The research problems provide suitable material for the training of postdoctral associates and graduate and advanced undergraduate students.

Findings:

The main result of the PI (joint with student Serguei Norine) is the discovery of an infinite sequence of reducible configurations for the Four-Color Theorem.

Sang-il Oum (with P. Seymour) found a general polynomial-time algorithm that for every fixed integer k, given an integer-valued symmetric submodular function f as input outputs either a branch-decomposition of width at most k or a valid statement that no such branch-decomposition exists. This applies to branch-width of matroids and rank-width of graphs, and in both cases this is the first known polynomial-time algorithm.

Oum also worked on approximation algorithms to approximate rank-width. The only previous algorithm (by Oum and Seymour) runs in O(n^9log n), but it works for more general branch-width and so was not optimized for rank-width. Oum now has two algorithms, a purely graph-theoretic one that runs in O(n^4) time and an O(n^3) algorithm based on matroid branch-width.

With J. Geelen, Oum obtained an interesting class of delta-matroids called graphic delta-matroids. Using this, they were able to show that line graphs of sufficiently large rank-width must contain a fixed bipartite circle graph as a pivot-minor.

Serguei Norine, while a postdoc at Georgia Tech, in joint work with M. Baker obtained an analogue of the Riemann-Roch theorem for graphs, which then led to a Riemann-Roch theorem for tropical curves, and other results that explored the analogies between Riemann geometry and graph theory.

Training and Development:

The research on this project contributed to the professional development of my graduate student Serguei Norine. Serguei received his PhD from Georgia Tech in 2005 and took up a highly paid position as a financial analyst with D. E. Shaw & Co. After

a year on Wall Street he decided he preferred to be a mathematician. I was able to hire him as a postdoctoral associate during the academic year 2006/07. In 2007 he turned down a tenure-track offer from the Depatment of Combinatorics and Optimization at the University of Waterloo to take up a Veblen Instructorship at Princeton University.

For years two and three I recruited Sang-il Oum as a postdoctoral associate, who continued his research program on branch-width of graphs and matroids, clique-width of graphs and applications to algorithms. Sang-il left early to take up a 1.5 year position in the Depatment of Combinatorics and Optimization at the University of Waterloo, and is currently negotiating an Assistant Profesorship at the Korean Institute for Science and Technology.

Since Oum left his position early I was able to recruit Zdenek Dvorak for one year as a postdoc. Upon completing his term at Georgia Tech Zdenek took up another postdoctoral appointment at Simon Fraser University.

Undergraduate REU students (not supported by this grant):
Matthew Perry and Luke Snyder

PhD students:
Zixia Song, PhD 2004
Serguei Norine, PhD 2005
Paul Wollan, PhD 2005
Rajneesh Hegde, PhD 2006
Torsten Inkmann, PhD 2008
Luke Postle, PhD in progress
Carl Yerger, PhD in progress
Arash Asadi, PhD in progress

Outreach Activities:

I have lectured about my work on many occasions, including giving public lectures. In 2005 I have taught a minicourse at the Euler Institute at the Technical University of Eindhoven, Holland. In 2006 I gave an invited section talk at the International Congress of Mathematicians in Madrid. In 2007 I gave a tutorial at the Fall School on Algorithmic Graph Structure Theory at Schloss Blankensee, near Berlin, Germany. I maintain a web site about the Four-Color Theorem; see below. I have written a survey article about our work on perfect graphs, and a survey article about Pfaffian orientations for the International Congress in Madrid.

Journal Publications

S. Oum and P. Seymour, "Approximating clique-width and branch-width", J. Combin. Theory Ser. B, p. 514, vol. 96, (2006). Published,

S. Oum, "Rank-width and vertex-minors", J. Combin. Theory Ser. B, p. 79, vol. 95, (2005). Published,

B. Courcelle and S. Oum, "Vertex-minors, monadic second-order logic, and a conjecture by Seese", J. Combin. Theory Ser. B, p. 91, vol. 97, (2007). Published,

S. Oum, "Rank-width and well-quasi-ordering", SIAM J. Discr. Math., p. 666, vol. 22, (2008). Published,

S. Oum and P. Seymour, "Certifying large branch-width", Proceedings of the Seventeenth Annual ACM-SIAM Symposium on Discrete Algorithms (Miami, FL, 2006). SODA '06. ACM, New York, p. 810, vol. , (2006). Published,

S. Oum and P. Seymour, "Testing branch-width", J. Combin. Theory Ser. B, p. 385, vol. 97, (2007). Published,

S. Oum, "Approximating rank-width and clique-width quickly", Lecture Notes in Comput. Sci., p. 49, vol. 3787, (2005). Published,

P. Hlineny, S. Oum, D. Seese and G. Gottlob, "Width parameters beyond tree-width and their applications", The Computer Journal, p. 326, vol. 51, (2008). Published,

Norine, S; Thomas, R, "Minimally non-Pfaffian graphs", JOURNAL OF COMBINATORIAL THEORY SERIES B, p. 1038, vol. 98, (2008). Published, 10.1016/j.jctb.2007.12.00

Norine, S; Sereni, JS, "Graphs with full rank 3-color matrix and few 3-colorings", JOURNAL OF COMBINATORIAL THEORY SERIES B, p. 1115, vol. 98, (2008). Published, 10.1016/j.jctb.2007.12.00

Norine, S, "Pfaffian graphs, T-joins and crossing numbers", COMBINATORICA, p. 89, vol. 28, (2008). Published, 10.1007/s00493-008-2150-

Norine, S; Thomas, R, "Pfaffian labelings and signs of edge colorings", COMBINATORICA, p. 99, vol. 28, (2008). Published, 10.1007/s00493-008-2231-

Baker, M; Norine, S, "Riemann-Roch and Abel-Jacobi theory on a finite graph", ADVANCES IN MATHEMATICS, p. 766, vol. 215, (2007). Published, 10.1016/j.aim.2007.04.01

Ghebleh, M; Kral, D; Norine, S; Thomas, R, "The circular chromatic index of flower snarks", ELECTRONIC JOURNAL OF COMBINATORICS, p. , vol. 13, (2006). Published,

Norine, S; Seymour, P; Thomas, R; Wollan, P, "Proper minor-closed families are small", JOURNAL OF COMBINATORIAL THEORY SERIES B, p. 754, vol. 96, (2006). Published, 10.1016/j.jctb.2006.01.00

Norine, S, "Drawing Pfaffian graphs", GRAPH DRAWING, p. 371, vol. 3383, (2004). Published,

Norine, S; Little, CHC; Teo, KL, "A new proof of a characterisation of Pfaffian bipartite graphs", JOURNAL OF COMBINATORIAL THEORY SERIES B, p. 123, vol. 91, (2004). Published, 10.1016/j.jctb.2003.10.00

Norine, S, "On two questions about circular choosability", JOURNAL OF GRAPH THEORY, p. 261, vol. 58, (2008). Published, 10.1002/jgt.2030

Norine, S; Thomas, R, "Generating bricks", JOURNAL OF COMBINATORIAL THEORY SERIES B, p. 769, vol. 97, (2007). Published, 10.1016/j.jctb.2007.01.00

Books or Other One-time Publications

Web/Internet Site

URL(s):

http://www.math.gatech.edu/%7Ethomas/FC/fourcolor.html

Description:

The site pertains to work done under earlier NSF awards, but it is related to the current project because it continues to draw attention, and thus potentially contributes to human resource development, as described later in this report.

Other Specific Products

Contributions

Contributions within Discipline:

The work on the Four-Color Theorem addresses a 160 year old problem and works toward finding a computer-free proof.

Oum looks at several areas that were seldom discussed together, namely clique-width of graphs, matroid structure theory, and delta matroids and isotropic systems developed by Bouchet.

The work of Norine (some joint with the PI) contributed to the understanding of matching structure of graphs, Pfaffian orientations of graphs and their relations to other areas such as crossing numbers and edge-colorings.

Finally, the work of Baker and Norine made a substatial contribution to the emerging field that studies analogies of the classical Riemann surfaces and more modern topics of metrized graphs and more generally tropical curves.

Contributions to Other Disciplines:

Contributions to Human Resource Development:

Training of postdocs, graduate and undergraduate students.

Contributions to Resources for Research and Education:

I maintain a popular website (reported elsewhere) about the Four-Color theorem, its history and a new proof found with the support of an earlier NSF grant. Very often I receive various questions from individuals who visited the web site. Many of them are high school or college students, and I would hope that for at least some of them reading my website will generate further interest in pursuing a career in science.

Contributions Beyond Science and Engineering:

Categories for which nothing is reported:

Organizational Partners

Any Book

Any Product

Contributions: To Any Other Disciplines

Contributions: To Any Beyond Science and Engineering